Coastal Engineering
Technical Note

Irregular Wave Runup on Beaches

PURPOSE: To present a method for estimating the maximum irregular wave runup for natural beaches.

BACKGROUND: Relationships for predicting wave runup on structures as given in the Shore Protection Manual (SPM 1984) are based on small-scale monochromatic wave tests, with measurements of runup made on smooth, impermeable slopes. Monochromatic (or regular) waves, all of the same height and period, are best approximated in nature by swell from distant storms. Irregular seas, made up of many types of waves of different heights and periods traveling in different directions, are typically the conditions that occur in a random sea state, such as a local storm sea. The roughness and percolation qualities of sediment comprising a natural beach tend to reduce runup compared to the smooth, impermeable slope experiments on which the SPM is based. Also most natural beaches have milder slopes than the data in the SPM addresses.

The upper limit of wave runup at a site can be used as the upper elevation of a protective beach or dune system, if overtopping cannot be permitted. Estimations of runup can also be utilized to approximate the frequency and duration of wave uprush at a back-beach seawall or revetment. A method developed by Resio (1987) to predict the maximum irregular wave swash amplitude on natural beaches with sand ranging from 0.1 mm - 0.5 mm is presented herein. Because Resio's method does not include the effect of wave setup in the swash amplitude, a simple conservative relationship presented by Holman (1986) is used to calculate setup and add to the swash amplitude predicted by Resio's method, thereby obtaining the maximum irregular wave runup.

PREDICTION METHOD: Resio (1987) developed a method to estimate the maximum wave swash amplitude that will occur during a storm on natural beaches utilizing data collected by Holman (1986) at the Coastal Engineering Research Center's (CERC) Field Research Facility (FRF) in Duck, North Carolina. Resio supplemented Holman's data with a lower energy swash amplitude data set collected by Carlson (1984) on two beaches in San Francisco Bay, where waves were locally generated (no oceanic swell). The ranges of conditions encompassed by the two data sets are detailed in Table 1.

Resio evaluated the FRF swash amplitude data using wave information measured in 2 m, 8 m, and 17 m depths, and found that the swash amplitude data were best related to the wave conditions (wave height and length) measured at the 8 m depth. Out of 32 data sets collected by Holman, Resio fit a relationship to the "tail" (extreme values) of the swash amplitude distribution using the 15 largest swash amplitudes observed in each data set. Extremal swash

Table 1. Range of conditions for each data set utilized by Resio (1987).

Study	Median Grain Size (mm)	Foreshore Beach Slope	Wave Height (m)	Wave Period (sec)	Depth Waves Measured (m)	Surf Similarity Parameter (see Eqn (4))
Holman (1986)	0.3	0.07-0.20	0.4-4.0	6-16	8	0.5-4.0
Carlson (1984)	"fine to med" (0.1-0.5)	0.11,0.08	0.1-0.3	2,4	"out of surf zone"	0.5-0.9

amplitudes measured by Carlson (1984) were well predicted using the relationship, and Resio concluded that the technique appeared applicable for most natural sand beaches. The procedure predicts the expected extremal swash amplitude based on the number of waves that can be expected to occur during the storm event under consideration, N_s. A dimensionless variable y is given as a function of N_s as follows:

$$y = -\ln\left[\ln\frac{N_s}{N_s - 1}\right] \tag{1}$$

The variable y is related to a function $f(N_s)$; knowing y, this function can then be computed as:

$$f(N_s) = 0.20 + 0.20\left[\frac{1 - \exp(-0.19\,y)}{0.19}\right] \tag{2}$$

The relative swash amplitude $\hat{A}$ can then be found as follows.

$$\hat{A} = f(N_s)\,\xi \tag{3}$$

where the surf similarity parameter ξ is defined as

$$\xi = \frac{\tan\theta}{(H_{mo8} / L_p)^{1/2}} \tag{4}$$

and the elevation of maximum swash amplitude A is defined as

$$A = \hat{A}\,H_{mo8} \tag{5}$$

where θ = foreshore beach slope angle (defined to be positive);
H_{mo8} = zero-moment wave height at 8 m depth; and
L_p = wave length associated with peak spectral period in 8 m depth.

The relationship developed by Resio should be applicable for sites with plane, parallel offshore contours, with sand beaches with median grain sizes ranging from fine to medium sand (0.1 mm - 0.5 mm).

If wave data are not available at an 8 m depth, they can be transformed as an irregular sea to the 8 m depth utilizing a relationship developed by Hughes (1984) as follows:

$$\frac{H_{mo}}{H_{mo8}} = \left[\frac{L_{po}}{L_p} \right]^{3/4} \tag{7}$$

where H_{mo8} and L_p are as defined previously, and
H_{mo} = zero-moment wave height in deep water;
L_{po} = wave length associated with peak spectral period in deep water

$$= \frac{g\ T_p^2}{2\ \pi} ,$$

where
T_p = peak spectral wave period; and
g = acceleration of gravity (9.81 m^2/sec = 32.2 ft^2/sec)

For deep water wave conditions, the zero-moment wave height H_{mo} can be approximated by the significant wave height H_s (SPM 1984). Equation (7) is valid for the conditions of no refraction or diffraction and an equal and constant wind both in deep water and at the 8 m water depth (Hughes 1984).

Before the wave height in 8 m can be calculated using Equation (7), the wave length at an 8 m depth, L_p , must be known. To avoid an iterative solution in calculating the wave length at the 8 m depth, Table C-1 of the SPM (1984) or Hunt's method as discussed in CETN-I-17 using the peak spectral period T_p (U.S. Army Corps of Engineers 1985) can be utilized.

Wave setup $\bar{\eta}$, which should be added to the extremal runup to obtain a maximum expected water level, can be calculated using a conservative relationship that encompasses nearly all of the data presented by Holman (1986):

$$\bar{\eta} = \xi\ H_s \tag{8}$$

APPLICATION: The expected extremal swash amplitude plus wave setup at a sand beach can be estimated as follows:

a. Determine the desired storm duration t for which an extremal runup will be calculated (i.e, 3 hr, 6 hr, 12 hr storm).

b. Determine deep water wave height H_{mo} and peak spectral period T_p corresponding to the storm event (if wave conditions corresponding to the storm event are known at the 8 m depth, skip to step f).

c. Calculate wave length in deep water $L_{po} = g\,T_p^2/2\pi$

d. Calculate wave length L_p at the 8 m depth using the SPM or CETN-I-17.

e. Calculate wave height H_{mo8} at the 8 m depth utilizing the deep water wave information and the relationship presented in Equation (7) above.

f. Calculate the number of waves N_s occurring during the storm using the wave period T_p and storm duration t as follows:

$$N_s = \frac{t(hr)\ \ 60(min/hr)\ \ 60(sec/min)}{T_p\ (sec/wave)} \tag{9}$$

g. Calculate the function $f(N_s)$ corresponding to the number of waves occurring during the storm interval, N_s, using Equations (1) and (2). Equations (1) and (2) have been combined graphically, giving the relationship of the function $f(N_s)$ to the number of waves that occur during the storm of consideration N_s (Figure 1).

h. Calculate the extremal elevation of the swash amplitude A as follows:

$$A = f(N_s)\ \tan\theta\ \left(H_{mo8}\ L_p \right)^{1/2} \tag{10}$$

i. Using deep water significant wave height H_s (equal to H_{mo} in deep water), calculate estimate of setup using Equation (8) and add to extremal swash amplitude to obtain maximum irregular wave runup.

**********************************EXAMPLE**********************************

GIVEN: Median grain size of a proposed beach fill at a site is approximately 0.3 mm, which will be placed with a foreshore slope of 1:15. Wave Information Study (WIS) deep water wave conditions corresponding to a 3-hr storm event are $H_s = 5$ m, and $T_p = 10$ sec. For design purposes, assume the storm occurs during a spring high tide which, including storm surge, can be assumed to be at + 2.0 m relative to Mean Sea Level (MSL) for the t = 3-hr storm event. Offshore contours at the proposed site are plane and parallel, and wind conditions during a 3-hr duration storm are typically constant from deep water to the 8 m depth. Assume no refraction or diffraction from deep water to the 8 m depth.

PROBLEM. Determine the required berm elevation for the proposed beach fill such that the berm will not be overtopped during a 3-hr storm event at high tide.

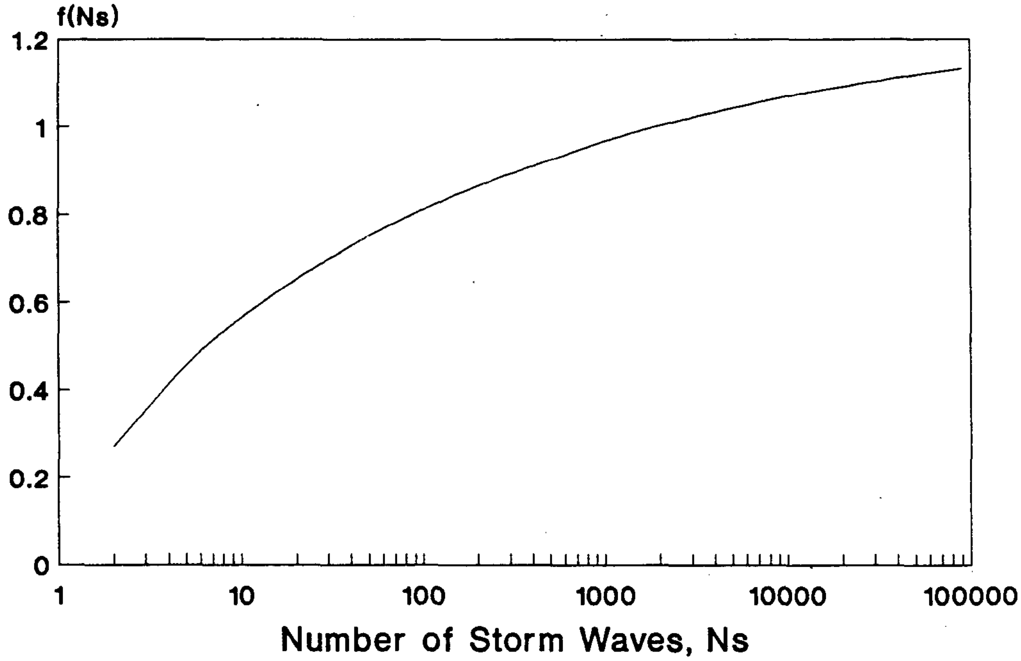

Figure 1. The relationship between $f(N_s)$ and the number of storm waves, N_s.

SOLUTION:

a. Given.

b. For deep water wave conditions, H_s is approximately equal to $H_{mo} = 5$ m; T_p is given = 10 sec.

c. Calculate deep water wave length.
$L_{po} = 9.81\ (10)^2/2\ \pi\ = 156.1$ m.

d. Calculate wave length in 8 m depth (CETN-I-17).
$G = (2\pi/10)^2\ (8/9.81) = 0.322$
$F = 0.322 + (1/(1 + 0.6522(0.322) + 0.4622(0.322)^2 + 0.0864(0.322)^4 + 0.0675(0.322)^5)$
$= 1.116$
$L_p = 10(9.81(8)/1.116)^{1/2} = 83.9$ m.

e. Calculate wave height in 8 m depth (Equation (7)).
$H_{mo8} = 5\ (83.9)^{3/4}\ /(156.1)^{3/4} = 3.13$ m.

f. Determine number of waves occurring during storm (Equation (9)).

$N_s = (3(60)(60))/10 = 1080$.

g. Using Figure 1, determine $f(N_s)$.

$f(N_s) \doteq 0.97$

h. Calculate elevation of swash amplitude (Equation (10)).

$A = 0.97\ (1/15)\ (3.13\ (83.9))^{1/2} = 1.1$ m.

i. Calculate surf similarity parameter (Equation (4)), with which setup can be calculated (Equation (8)).

$\xi = (1/15)\ (3.13/83.9)^{-1/2} = 0.35$

$\bar{\eta} = (0.35)\ (5) = 1.8$ m

Therefore, to avoid overtopping of the beach berm during a 3-hr storm event with 2.0 m tide and storm surge elevation, design the berm at an elevation equal to or greater than 1.1 m (extremal swash amplitude) + 2.0 m (tide and storm surge) + 1.8 m (setup) = 4.9 m MSL.

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